NONSTATIONARY HEAT TRANSFER IN A SCATTERING MEDIUM

Yu. A. Popov and V. I. Polovnikov

UDC 536.3+536.46

A numerical solution is given for nonstationary heat transfer in a bed showing thermal conduction, radiation scattering, and chemical heat production. The thermal-exposition situation is considered for a cloud of fine particles. It is found that the optical thickness of the medium affects the induction period.

Numerical studies have been made [1-4] of nonstationary heat transfer in scattering media; these concern a planar layer of gray thermally conducting material. There are isotropic treatments of the scattering [1, 2] and anisotropic ones [3, 4]. The difference of the present study from [1-3] is that a simple but reasonably accurate method is used to incorporate the scattering. It is assumed that the medium contains chemical heat sources. The walls are assumed to be black. The reaction rate is defined by an Arrhenius law.

The energy equation takes the following form [7] if the scattering is neglected:

$$\frac{\partial\Theta}{\partial\xi}=\frac{1}{2}\int_0^{\tau_0}\Theta^4(\tau')\,E_1(|\tau'-\tau|)\,d\tau'-\Theta^4(\tau)+\frac{\Theta_1^4E_2(\tau)+\Theta_2^4E_2(\tau_0-\tau)}{2}+N\frac{\partial^2\Theta}{\partial\tau^2}+A\exp\left[-\frac{E}{T_0\Theta(\tau)}\right],\qquad(1)$$

where $\Theta=T/T_0$, $\Theta_1=T_1/T_0$, and $\Theta_2=T_2/T_0$ are dimensionless temperatures; $\xi=(4\alpha\sigma T_0^3/\rho_0c_p\cdot t$ is dimensionless time; $N=\alpha\lambda/4\sigma T_0^3$ is a conductive-radiative parameter; $A=v_0q/4\alpha\sigma T_0^4$ is a dimensionless preexponential factor; $\tau=\alpha x$, and $\tau_0=\alpha l$ are the optical depth and thickness of the layer; and $E_n(\tau)$ is the integral exponential fucntion of order n.

The scattering is incorporated in a one-dimensional approximation, i.e., the scattering indicatrix is approximated as highly elongated (in the forward and backward directions). Table 1 compares the calculated reflectivity and transmission for a spherical scattering indicatrix in the one-dimensional approximation with the exact solution derived by the method of [5]. Table 2 gives a comparison of the transmission for the actual scattering indicatrix. The form of the indicatrix was taken from [9] with $\rho=1$ and $n=2$. The calculation was performed in the one-dimensional approximation by the method of moments [10], whose accuracy is greater. The structure of (1) is not altered when the scattering is incorporated in the one-dimensional approximation, but the kernel of the equation and E_2 takes more complicated forms. The absorption coefficient is replaced by the attenuation coefficient in the expressions for ξ, N, A, τ, and τ_0; we use an expression for the divergence of the radiation flux-density vector in this approximation [8] to obtain

$$\frac{\partial\Theta}{\partial\xi}=(1-\gamma)\left\{\frac{(1-\gamma)}{2}\left[\int_0^{\tau}\Theta^4(\tau')\,E_1^*(\tau',\,\tau)\,d\tau'+\int_{\tau}^{\tau_0}\Theta^4(\tau')\,E_1^*(\tau_0-\tau',\,\tau_0-\tau)\,d\tau'\right]-\right.$$

$$\left.-\Theta^4(\tau)+\frac{\Theta_1^4E_2^*(0,\,\tau)+\Theta_2^4E_2^*(0,\,\tau_0-\tau)}{2}\right\}+N\frac{\partial^2\Theta}{\partial\tau^2}+A\exp\left[-\frac{E}{T_0\Theta(\tau)}\right].\qquad(2)$$

The functions E_n^* take the form

$$E_n^*(\tau',\,\tau)=\int_0^{+1}\mu^{n-2}\,I_0(\tau')\,\frac{e^{-\frac{\varkappa(\tau-\tau')}{\mu}}}{1-R_\infty^2e^{-\frac{2\varkappa(\tau_0-\tau')}{\mu}}}\times$$

$$\times\left[1-R_\infty^2e^{-\frac{2\varkappa(\tau_0-\tau)}{\mu}}+R_\infty-R_\infty e^{-\frac{2\varkappa(\tau_0-\tau)}{\mu}}\right]d\mu,$$

All-Union Scientific-Research Institute of Metallurgical Heat-Engineering, Sverdlovsk.
Transated from Inzhenerno-Fizicheskii Zhurnal, Vol. 34, No. 6, pp. 1090-1095, June, 1978. Original article submitted May 30, 1977.

 0022-0841/78/3406-0738$07.50

TABLE 1. Comparison of the Exact Solution for the Transmission D and Reflectivity R of a Layer with a Spherical Scattering Indicatrix with the Solution in the One-Dimensional Approximation for a Layer Receiving Hemispherical Radiation

γ	$\tau_0=0,5$				$\tau_0=1$			
	R	R_q	D	D_q	R	R_q	D	D_q
0,2	0,0373	0,0417	0,4741	0,4783	0,0441	0,0514	0,2462	0,2526
0,4	0,0818	0,0894	0,5125	0,5190	0,0999	0,1137	0,2829	0,2949
0,6	0,1366	0,1453	0,5603	0,5670	0,1743	0,1919	0,3360	0,3504
0,8	0,2057	0,2125	0,6219	0,6254	0,2802	0,3954	0,4162	0,4276
γ	$\tau_0=2$				$\tau_0=3$			
	R	R_q	D	D_q	R	R_q	D	D_q
0,2	0,0462	0,0552	0,0728	0,0783	0,0463	0,0557	0,0228	0,0260
0,4	0,1067	0,1250	0,0925	0,1049	0,1073	0,1267	0,0317	0,0398
0,6	0,1920	0,2185	0,1268	0,1466	0,1944	0,2237	0,0496	0,0651
0,8	0,3279	0,3559	0,1972	0,2119	0,3386	0,3734	0,0957	0,1197

TABLE 2. Transmission D of a Layer of Nonabsorbing Particles H Having Size Parameter $\rho = 1$ and Refractive Index $n = 2$ ($\bar{\mu} = 0.277$, $\gamma = 1$)

τ_0	D		τ_0	D	
	method of moments	one-dimensional approximation		method of moments	one-dimensional approximation
0,2	0,88	0,88	1,4	0,60	0,55
0,4	0,80	0,80	1,6	0,58	0,52
0,6	0,74	0,73	1,8	0,55	0,49
0,8	0,69	0,67	2,0	0,52	0,46
1,0	0,65	0,62	2,2	0,50	0,44
1,2	0,62	0,58	2,4	0,48	0,42

where

$$\varkappa = \sqrt{(1-\gamma)(1-\bar{\mu}\gamma)},$$

$$R_\infty = \frac{\sqrt{1-\bar{\mu}\gamma}-\sqrt{1-\gamma}}{\sqrt{1-\bar{\mu}\gamma}+\sqrt{1-\gamma}},$$

$$I_0(\tau') = \frac{1+R_1}{1-R_1R_2},$$

$$R_1 = \frac{R_\infty\left(1-e^{-\frac{2\varkappa\tau'}{\mu}}\right)}{1-R_\infty^2 e^{-\frac{2\varkappa\tau'}{\mu}}}, \quad R_2 = \frac{R_\infty\left(1-e^{-\frac{2\varkappa(\tau_0-\tau')}{\mu}}\right)}{1-R_\infty^2 e^{-\frac{2\varkappa(\tau_0-\tau')}{\mu}}}.$$

Equation (2) was solved numerically by standard methods [7]; an explicit difference scheme was used. The boundary condition was that the wall temperature was constant. It was assumed that the temperature of the medium was independent of the coordinate at the start. The temperature at step m + 1 in time was determined from the temperature distribution in step m. The temperature distribution at step m was substituted on the right in (2) and the expression was multiplied by $\Delta\xi$ to find the temperature increment. The new temperature distribution was used to define the temperature increment for step m + 1. Then the temperature change for step m was corrected, which gave a more accurate temperature distribution for step m + 1. The integral in (2) was derived by Simpson's method, while the E_n^* were calculated by Gauss quadrature. The number of steps required for the thickness of the layer and for the time was determined from the condition that the solutions should agree when the number of divisions is doubled.

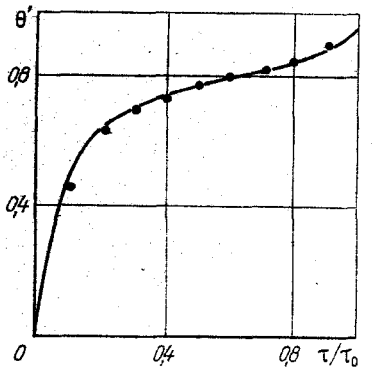

Fig. 1

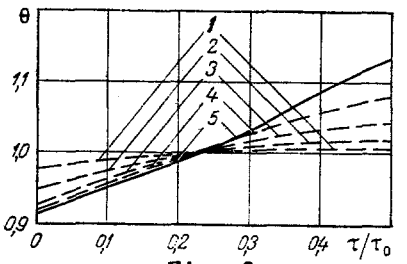

Fig. 2

Fig. 1. Distribution of the dimensionless temperatures for $\xi = 0.25$ and $\Theta' = (T - T_1)/(T_2 - T_1)$; the solid line is from [1], while the points are from our calculation.

Fig. 2. Temperature distributions during the development of a thermal explosion for a nonscattering layer: 1) $\xi = 0.10$; 2) 0.20; 3) 0.26; 4) 0.30; 5) 0.32. $\xi_i = 0.33$.

The accuracy was evaluated by comparing the results with the data of Weston and Hauth [1] for isotropic scattering; in that case, the contribution from radiation was the largest. The calculation was performed with the following parameters: $\tau_0 = 1$, $\xi = 0.25$, $T_0 = T_1$, $\Theta_1 = 1$, $\Theta_2 = 2$, $N = 0.04$, $\mu = 0$, $\gamma = 0.5$, and $A = 0$. Figure 1 shows the comparison. It is clearly possible to use this method of correcting for the scattering.

The chemical heat sources may cause thermal explosion; this is a topic that has not so far been considered in the literature. We have performed numerical calculations on thermal explosion in a plane-parallel cloud of particles suspended in air. The definitive temperature was the initial temperature of the medium: $T_0 = 770°K$. In the nonscattering case, the calculations were performed for various τ_0 with the following values of the parameters: $\Theta_1 = \Theta_2 = 0.39$, $N = 0$, and $A = 0.61 \cdot 10^{11}$. The induction period was defined as the period from the start of the process to the instant of sharp rise in temperature at the center of the layer, which was the point at which the computer halted on account of overflow. The error in determining the induction time in this way was about 1%, since the scheme describes the approach to thermal explosion satisfactorily but does not describe the explosion itself, since the latter occurs in a time short in comparison with the induction period. Figure 2 shows temperature distributions for this case with τ_0 of 1 and 2. Figure 3 shows that the optical thickness has a considerable effect on the induction period. The effect is particularly large for $1 \leq \tau_0 \leq 5$ with these parameters, while for $\tau_0 \leq 1$ the layer cools, and for $\tau_0 \geq 5$ the induction period approaches the value for adiabatic conditions asymptotically. The induction period is reduced by raising the temperature of the medium or of the walls.

The calculation was performed with scattering for carbon particles having a size parameter ρ of 4-10 (see [6] on the optical constants of carbon particles). It can be assumed as an approximation that $k_\alpha = 1$, $\gamma = 0.63$ in this range of ρ; the mean value $\mu \approx 0.8$ was computed from Mie's theory. The linear attenuation coefficient is related to the particle concentration:

$$\beta = kc \frac{f}{4} .$$

An interesting point is that A is independent of the particle concentration when all the definitive quantities have been inserted. The concentration affects the result only via the optical thickness and depth. Figure 4 shows temperature distributions calculated for a scattering layer with the following parameters: $\tau_0 = 3.24$, $\Theta_1 = \Theta_2 = 0.39$, $N = 0$, $A = 0.226 \cdot 10^{11}$, $\mu = 0.8$, and $\gamma = 0.63$. These have been chosen in such a way as to correspond to the product of the absorption coefficient and layer thickness used in the nonscattering case (Fig. 2). The temperature distribution was not qualitatively altered, although the induction period was very much shorter. This was due to the choice of τ_0 in the range where the optical thickness has considerable influence on the induction period. The effect of scattering is slight where the influence of the optical thickness is small.

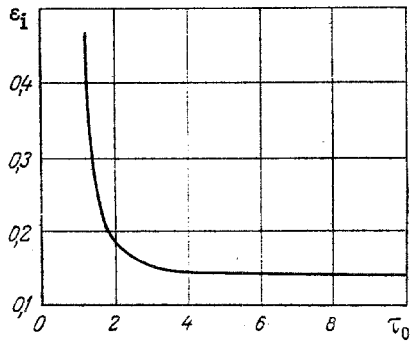

Fig. 3

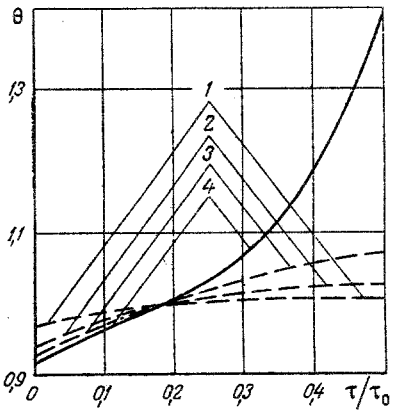

Fig. 4

Fig. 3. Effects of optical thickness in a nonscattering layer on the induction period.

Fig. 4. Temperature distributions during the development of a thermal explosion for a scattering layer: 1) ξ = 0.104; 2) 0.192; 3) 0.252; 4) 0.296; ξ_i = 0.30. (The dimensionless time has been determined by means of the absorption coefficient.)

NOTATION

T, T_1, T_2, temperatures of medium and of first and second walls; T_0, controlling temperature; α, linear absorption coefficient; β, linear attenuation coefficient; γ, single-scattering factor; σ, Stefan's constant; ρ_0, density; c_p, specific heat at constant pressure; t, time; λ, thermal conductivity; v_0, preexponential factor; q, heat of reaction; x, linear coordinate; l, layer thickness; R, reflectivity; D, transmissivity; τ, optical depth; τ_0, optical thickness; E, activation energy divided by the universal gas constant; R_∞, reflectivity of semiinfinite medium; k_α, dimensionless absorption coefficient; k, dimensionless attenuation coefficient; f, specific surface of particles; c, particle concentration; ρ, dimensionless ratio of particle diameter multiplied by π to wavelength; μ, mean cosine of the angle of scattering at a particle; n, refractive index. Indices: q, quasi-one-dimensional approximation.

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